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The performance of hybrid models in the assessment of default risk

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ABSTRACT

This paper combines fundamental analysis and contingent claim analysis into a hybrid model of credit risk measurement. Our database consists of French companies listed on the Paris Stock Exchange (Euronext Paris). Our objective is to assess how the combination of continuous assessments provided by the market and the values derived from financial statements improve our ability to forecast the default probability. During the first phase, the default probability is estimated using both methods separately, and subsequently, the default probability of the structural model is integrated at each point in time in the non-structural model as an additional explanatory variable. The appeal of the hybrid model allows the default probability to be continuously updated by integrating market information via the probabilities of default extracted from the structural model. Our results indicate that default probabilities extracted from the structural model contribute significantly in explaining default risk when included in a hybrid model with accounting variables.

1. Introduction

Credit risk refers to the risk due to unpredicted changes in the credit quality of a counter party or issuer and its quantification is one of the major frontiers in modern finance. The creditworthiness of a potential borrower affects the lending decision and the credit spread, since it is uncertain whether the firm will be able to perform its obligation. Credit risk measurement depends on the likelihood of default of a firm to meet its required or contractual obligation and on what will be lost if default occurs. When we consider the large number of corporations issuing fixed income securities and the relatively small number of actual defaults might regard default as rare event. However, all corporate issuers have some positive probability of default. Models of credit risk measurement have focused on the estimation of the default probability of firms, since it is the main source of uncertainty in the lending decision. We may distinguish two large classes of credit risk models. The first class of traditional models assumes the fundamental analysis, called the non-structural models. The goal of these models that goes back to Beaver (1966) and Altman (1968) is to find significant factors in assessing the credit risk. The second class, called structural models assumes the contingency claim analysis. The models refer to Black and Scholes (1973) and Merton (1974) and assume corporate liabilities as contingent claims on the assets of the firm.[4]

This paper investigates the hybrid contingent claim approach with French companies listed on the Paris Stock Exchange (Euronext Paris). The main objective is to assess how the combination of continuous assessments provided by the market and the values derived from financial statements improve our ability to forecast the probability of default.

The structural model of Merton has the advantage of being flexible, since the probability of default can continually be updated with changes in the value of corporate assets. Its main drawback is that it may over-or underestimate the probability of default, since asset values are unobservable and must be extrapolated from the share prices. On the other hand, the non-structural model of Altman is more accurate because it uses the accounting data of companies, but it is less flexible. Because the frequency of information is generally annual, the probabilities of default cannot be updated during the fiscal year. The quarterly financial statements can be found, but they are not always audited by an external accounting firm.

The Bank of England estimated the hybrid model with data from British companies and found some interesting results. During the first phase, the probability of defaults is estimated using both methods separately, and subsequently, the probability of default of the structural model is integrated at each point in time in the non-structural model as an additional explanatory variable. The appeal of the hybrid model

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[4] Another widely used category of credit risk models is the reduced form approach where the dynamics of default are given exogenously by an intensity or compensator process.

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allows the probability of default to be continuously updated by integrating market information via the probabilities of default extracted from the structural model. In this paper, we apply the hybrid model to French companies listed on Paris Stock Exchange (Euronext Paris).

This paper is organized as follows. Section 2 reviews the main models in the literature. Section 3 presents the estimated structural model and describes the data used. Section 4 presents the estimation of the hybrid model and summarizes the main results.

2. The main models for default risk assessment

2.1. Non-structural models

Traditional non-structural models adopt fundamental analysis and try to find which factors are important in explaining the credit risk of a company. They assess the significance of these factors, mapping a reduced set of financial ratios, accounting variables and other information into a quantitative score. The latter, can be interpreted as a probability of default and can be used as classification system.[5]

Beaver (1966) introduced the univariate approach of discriminant analysis in the default risk of firm's explanation. Altman (1968) has extended it to a multivariate context and developed the Z-Score model. It weights the independent variables (financial ratios and accounting variables) and generates a single composite discriminant score. Altman et al. (1977) have developed the ZETA model, which integrated some improvements to the original Z-Score approach. Then, the binary dependent variable models, known as the logit and probit models have been used in bankruptcy prediction.[6] Ohlson (1980) used logit methodology to derive a default risk model known as O-Score. Probit (logit) methodology weights the independent variables and allocates scores in a form of failure probability using the normal (logistic) cumulative function.

Binary credit risk models are used by banks for their non-listed firm lending procedure. Several banks use this method for privately and publicly traded companies, either by buying a model, such as RiskCalc Moody's, or by programming their own estimate. One problem they often face is to build an appropriate proper database. Very often, credit files are not computerized or do not contain historical data.

The main advantage of non-structural models is their accuracy in estimating probabilities of default. In addition, they are easy to use for financial institutions equipped with solid management systems of database and may produce very accurate default probabilities. Nonetheless, these models are not flexible, because they need information from financial statements. Thus, it is very difficult to update the probabilities of default over a year. Some financial institutions may require reporting on a quarterly basis, but they are rarely audited by accounting firms.

2.2. Structural models

The original Merton model is based on some simplifying assumptions about the structure of the typical firm's finances. The event of default is determined by the market value of the firm's assets in combination with the liability structure of the firm. When the value of the assets falls below a certain threshold, the firm is considered to be in default. The main criticism that leveled at Merton's model is that it does not account for the possibility that the firm may default before the debt matures. To improve this basic model, several extensions have been suggested in the literature.

Crosbie and Bohn (2003) summarize KMV's default probability model. KMV's default probability model is based on a modified version of the Black–Scholes–Merton framework in the sense that KMV allows default to occur at any point in time and not necessarily at the maturity of the debt. In this model multiple classes of liabilities are modeled. There are essentially three steps in the determination of the default probability. The first step is to estimate the market value and volatility of the firm's assets, the second step is to calculate the distance-to-default, the number of standard deviations the firm is away from default, and the third step is to transform the distance-to-default into an expected default frequency (EDF) using an empirical default distribution.

Brockman and Turtle (2002) propose using barrier options. Thus, rather than stockholders who wait for the debt to mature before exercising a standard European call option, we have a down-and-out option on the assets in which lenders hold a portfolio of risk-free debt and a short put option combined with a long down-and-out call option on the firm's assets. The last part gives them the right to place the company into bankruptcy when they anticipate that its financial health can only deteriorate. Wong and Choi (2006) demonstrate that estimating the parameters of the Brockman and Turtle (2002) model by maximum likelihood yields results that resemble those from the iterative estimation method used in this literature when the theoretical model is Merton's. The appeal of the maximum likelihood method is that it allows for statistical inference or, more specifically, calculating descriptive statistics for the estimated parameters, such as the value of the firm.

Tudela and Young (2005) present an application of the hybrid model. This application uses barrier options with a down-and-out call option. The authors estimate various models on data from non-financial English firms for the period 1990–2001. They use data on firms that did, and did not, default, for their estimates of probabilities of default in the structural model. First, they verify whether the two firm types represent different predicted probabilities of default. Second, they compare their hybrid model with other non-structural models to verify whether the additional probability of default (PD) variable is significant for explaining probabilities of default. Third, they measure the performance of their model with power curve and accuracy ratio type instruments.

3. Estimation of the probabilities of default with the structural model: application of the Tudela and Young Model (2005)

3.1. Model description

In this model, the authors use the theory of barrier options[7] and more precisely the call option down-and-out, which vanishes when the underlying asset reaches the barrier. We assume that the capital structure consists exclusively of debt and equity. The level of debt is denoted by B and (T-t) represents the time remaining to maturity of the debt, the value of the firm is At and the value, at time t, of the debt maturing at time T is V (A, T, t). The share value at time t is f (A, t). The total value of the firm at time t is:

$$A_t = V(A, T, t) + f(A, t). \tag{1}$$

To derive the probability of default using a barrier option we assume that the value of the firm's underlying assets follows the following stochastic process:

$$dA = \mu_A A dt + \sigma_A A \; dz \tag{2}$$

where $dz = \varepsilon\sqrt{dt}$ and $\varepsilon \sim N\,[0, 1]$.

[5] For a review of traditional models (Jones (1987); Caouette et al. (1998), Saunders (2002)).

[6] Jones (1987) concludes that binary dependent variable models do not lead to notable improvements in the predictive power of fundamental analysis when compared to the earlier LDA models.

[7] Other equity-based models of credit risk that use the concept of barrier options are Black and Cox (1976), Longstaff and Schwartz (1995) and Briys and de Varenne (1997).

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Assume that the firm's liabilities L are the sum of short-term plus one-half of long-term liabilities. Assume that L follows a deterministic process:

$$dL = \mu_L \; L \; dt. \tag{3}$$

We note the asset-liability ratio by k:

$$k = A/L \tag{4}$$

A default occurs when k falls below the default point called $\widetilde{k}$ at any time. To estimate the probability of default, we model how k changes over time. Differentiate Eq. (4) and use Eqs. (2) and (3) gives:

$$dk = (\mu_A - \mu_L)k \; dt + \sigma_A k \; dz. \tag{5}$$

We define: $\mu_A - \mu_L = \mu_k$ and $\sigma_A = \sigma_k$. The values μ_k and σ_k are needed to calculate the probabilities of default. Maximum likelihood techniques are used to obtain estimates of those two parameters, but to build the maximum likelihood function, we need first to derive an expression for the density function of k.

Given Eq. (5), we can derive the density function of $\ln\left(\frac{k_T}{k_t}\right)$. The defective density function is given by:

$$h\left(\ln\left(\frac{k_T}{k_t}\right)\right) = \frac{1}{\sqrt{2\pi \; \sigma_K^2 (T-t)}} \left\{ \exp\left[-\frac{\left(\ln\left(\frac{k_T}{k_t}\right) - \left(\mu_k - \frac{\sigma_k^2}{2}\right)(T-t)\right)^2}{2\sigma_k^2 (T-t)} \right] - \exp\left[\frac{2\ln\left(\frac{\widetilde{k}}{k_t}\right) \; \mu_k - \frac{\sigma_k^2}{2}\Big)}{\sigma_k^2} - \frac{\left(\ln\left(\frac{k_T}{k_t}\right) - 2\ln\left(\frac{\widetilde{k}}{k_t}\right) - \left(\mu_k - \frac{\sigma_k^z}{2}\right)(T-t)\right)^2}{2\sigma_k^2 (T-t)} \right] \right\}. \tag{6}$$

Eq. (6) is the probability density of not crossing the barrier and being at the point $\ln(k_T/k_t)$ ln at time T. It is used to construct the likelihood function to be maximized in order to obtain estimates of μ_k and σ_k. These estimates are used to calculate the probability of default. The probability of the firm not defaulting until date T is given by the probability of $k_T > \widetilde{k}$ conditionally $k_\tau > \widetilde{k} \quad \forall \tau \;\; t \leq \tau \langle \; T => $

$$PD = 1 - \{[1 - N(u_1)] - \overline{w}[1 - N(u_2)]\} \quad \text{Where}: \; \ln\frac{\widetilde{k}}{k_t} = \widetilde{K}$$

$$u_1 = \frac{\widetilde{K} - \;\; \mu_k - \frac{\sigma_k^2}{2}\Big)(T-t)}{\sigma_k \sqrt{T\text{-}t}} \quad u_2 = \frac{-\widetilde{K} - \;\; \mu_k - \frac{\sigma_k^2}{2}\Big)(T-t)}{\sigma_k \sqrt{T-t}}$$

$$\overline{w} = \exp\left[\frac{2\widetilde{K}\left(\mu_k - \frac{\sigma_k^2}{2}\right)}{\sigma_k^2} \right]$$

and N(.) is the cumulative probability function of the normal distribution. For a European call option, the probability of default is $N(u_1)$. For the barrier option we see that the term $\overline{w}\,[1 - N(u_2)]$ adjusts the probability of default to take into account that the firm can default before the horizon date T. The Bank of England set $\widetilde{k} = 1$. We shall adopt this normalization. We assume that the ratio, $y = X/L$ where X represents the market capitalization of the firm and L is its liabilities as a proxy for the ratio $k = A/L$ since the value of the firm's assets is unobservable.

We use Matlab to estimate μ_k and σ_k with the maximum likelihood method, then we calculate the probabilities of default. Parameters μ_k and σ_k are estimated on the basis of a 24-month window for all firms. (As starting value we take $\sigma_k = 0.4$ and $\mu_k = 0.3$). Tudela and young add some accounting variables in their model to increase the model performance slightly. The final model of the Bank of England is as follows:

> PD = f [probability of default (1–2 years), profitability, debt over assets, cash over liabilities, sales growth, log number of employees, GDP].

The authors have applied this model to calculate the probability of default on data from non-financial English firms. We apply it to a sample of French listed companies but retaining other explanatory variables from the hybrid model.

3.2. Data

This section presents the data and explains how we calculate probabilities of default. This data is used also to estimate the hybrid model in Section 4. Our initial database contains 20 companies that did not default and 14 companies that did. The study period for the probabilities of default is from January 2004 to December 2005. The methodology to compute the probabilities of default with the structural model requires that our data window extends 24 months prior to the estimation period for the predicted probabilities of default in order to ensure statistical reliability. Market capitalization has a monthly frequency while the values of debt are observed annually. Thus the value of debt is considered during the year. We tried with data relevant to other periods but the problem is that firms defaulting change and we could not have a stable data over several periods. Therefore, we restrict our analysis to an accounting year.

3.2.1. Companies that have defaulted

Data on companies that have defaulted are from DIANE. However, 6 companies that defaulted were removed from the database because of lack of data (accounting and/or market) or because too large shift between the date of publication of the last financial statement and effective date of default. Indeed, companies have significant gaps between these two dates. This is explained by the fact that most of the firms do not publish their financial statements during the last year prior to bankruptcy. Another explanation is the slow process of putting in default of certain companies. Thus we eliminated firms with a lag of more than 18 months.

3.2.2. Companies that did not default

Accounting data on companies that did not default for the year 2005 and the monthly market capitalizations are extracted from the Diane Database.

3.2.3. Various statistics

Financial firms were eliminated from the database because they do not generally have the same structure of financial statements as non-financial firms. Thus the final database contains a total of 23 non-financial companies, 8 of them have defaulted. The following table presents the descriptive statistics of firms retained for analysis.

3.3. Estimation results

KMV's default probability model is based on a modified version of Merton's model in the sense that KMV allows default to occur at any point in time. Multiple classes of liabilities are used. Three steps are used in the determination of the default probability. The first step is to estimate the market value and volatility of the firm's assets. The second step is to calculate the distance to default, the number of standard deviations the firm is away from default. The last step is to transform the distance to default into an expected default frequency, EDF. Estimating probabilities from the structural model follows exactly the methodology reported on the website of KMV, which is also used in the industry and included in the software by Moody's KMV. It relies

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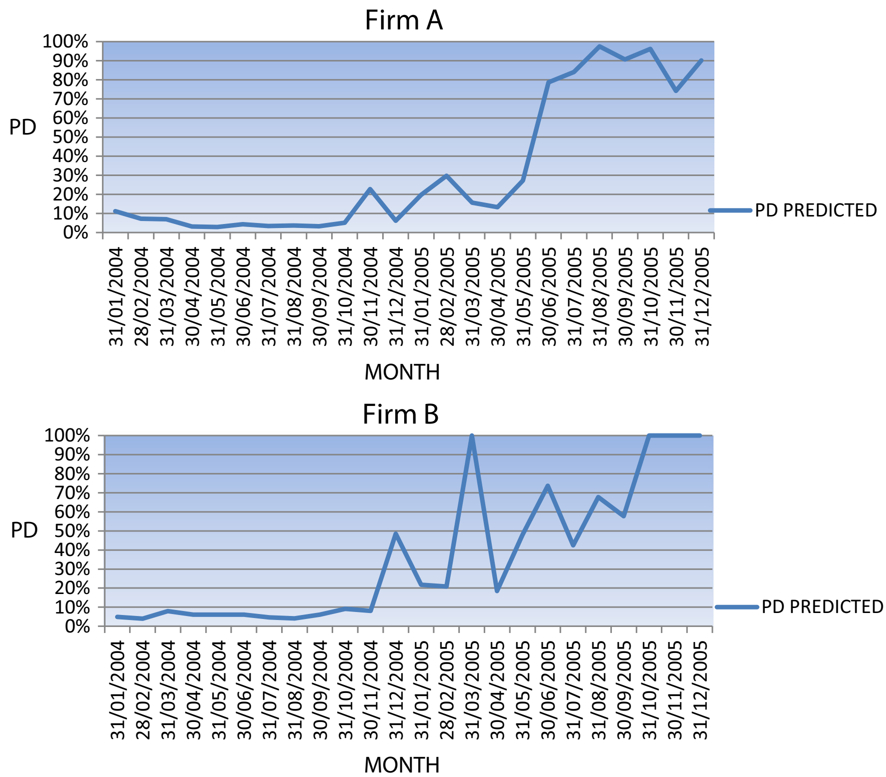

Fig. 1. Monthly default probabilities (2 years) of defaulting firms.

mainly on Merton's model. Estimating probabilities of default by the structural model provides the following results: for companies that have defaulted, the mean of probabilities of default is 33.97% while for companies that did not default it is 13.54%. The following figures show the evolution of the probabilities of default predicted for several firms. Fig. 1 shows the evolution of the probabilities of default for the firms that have defaulted.

4. The hybrid model

The main objective of this section is to check whether combining the structural and the non-structural models into a hybrid model yields a better measure of the default risk than those obtained from structural and traditional non-structural models estimated separately.

4.1. Methodology

We did not estimate the model with a simple linear regression, since we know that it must reflect non-linear behavior of the explanatory variables for defaults. In addition, it is well documented that simple linear models are inappropriate when the dependent variable is a probability. This model has the advantage of being easy to estimate but it has the disadvantage that it leads to PDs estimated to be out of the interval [0,1]. Thus, we must use other models, which keep the probability of default (PD) in the considered interval. This is particularly the probit model. In this type of model, the dependent variable is a dichotomous variable taking the value of 1 if an event occurs and 0 otherwise. In our case, the variable Y_i assumes the following values:

Y_i 1 if firm I defaults, and
Y_i 0 otherwise.

The vector of explanatory variables (financial ratios and accounting variables…) for firm *i* is denoted as X_i, while β is the vector of weights of these variables. The probit model assumes that there is a qualitative response variable (Y_i^*) defined by the following equation:

$$Y_i^* = \beta' X_i + \varepsilon_i. \tag{7}$$

In practice Y_i^* is an unobservable latent variable. We rather "observe" a dichotomous variable Y_i such that:

$$\begin{aligned} &Y_i = 1 \ \text{ if } \ Y_i^* > 0; \\ &Y_i = 0 \ \text{ otherwise.} \end{aligned} \tag{8}$$

In this form, $\beta' X_i$ is not $E(Y_i/X_i)$ as in the simple linear model, but rather $E\,(Y_i^*/X_i)$.

From Eqs.(7) and (8), we get

$$\text{Prob}(Y_i = 1) = \text{Prob}\ \ (\varepsilon_i > -\beta' X_i) = 1 - F(\beta' X_i) \tag{9}$$

where F is the cumulative distribution function of ε_i.

The functional form of F depends on the retained assumptions regarding the distribution of the residual errors (ε_i) in Eq. (7). The probit model is based on the assumption that these errors are independently

Table 1
Descriptive statistics of all firms retained for analysis (in million euros).

Statistic	Market value	Liabilities	Market value	Liabilities
	No default	No default	Default	Default
Mean	130,48	44,924	38,292	23,193
Median	101,128	38,714	17,073	9248
Maximum	386,65	156,147	190,854	120,568
Minimum	27,65	3955	11,473	7492
Standard deviation	94,013	36,76	61,698	39,383
Skewness	1464	1829	2259	2,2591
Kurtosis	4,7749	6,7184	6122	6,1207
Number of observations	360	30	180	16

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Table 2
Analysis of the maximum-likelihood estimators.

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5
Constant	−4.2571 (0.0245)	0.6808 (0.2001)	−7.91 (0.3719)	0.7242 (0.4283)	−3.8512 (0.2486)
PD (2 years)	0.1506 (0.0226)		0.2934 (0.3252)		0.1571 (0.1886)
Profitability		−0.0405 (0.1079)	−0.0229 (0.4589)		
Turnover		−0.0161 (0.0518)	−0.0213 (0.1988)	−0.0069 (0.4615)	−0.0164 (0.3557)
Equity/total assets				−0.0394 (0.0659)	−0.00709 (0.8033)
Debt/equity				0.0029 (0.8891)	0.0055 (0.8328)
Number of observations	23	23	23	23	23
Number of default	8	8	8	8	8
McFadden's R squared	0.5256	0.5934	0.8277	0.6137	0.7143
Likelihood ratio	15.6209 <0.0001	17.6367 0.0001	24.6009 <0.0001	18.2408 0.0003	21.2300 0.0002
Log likelihood	−7.0496	−6.0416	−2.5596	−5.7396	−4.2450

*Into parenthesis is the p-value of estimated parameters.

and identically distributed (i.i.d.) and follow a standard normal distribution N(0,1). The functional form can be written as:

$$F(-\beta' X_i) = \int_{-\infty}^{-\beta' X_i} \frac{1}{(2\pi)^{\frac{1}{2}}} \exp\left[-\frac{t^2}{2}\right] dt. \quad (10)$$

In this case, the observed values Y_i are simply the realizations of a binomial process whose probabilities are given by Eq (9) and vary from one observation to the next (with X_i). The likelihood function can be defined as follows:

$$l = \prod_{Y_i=0} F(-\beta' X_i) \prod_{Y_i=1} (1-F(-\beta' X_i). \quad (11)$$

And the parameter estimates β are those that maximize .

4.2. Variable selection

The objective is to verify if the combination of the structural and the non-structural models into the hybrid model represents a better measure of the default risk than structural and traditional non-structural models estimated separately. We explain default deficiencies by estimating a probit model in which the explanatory variables are the estimated probabilities of default from the structural model, financial ratios and other accounting data. The dependent variable is binary taking the value of 1 if the default occurs and 0 otherwise. Using the same methodology, we also estimate a model with only accounting data as explanatory variables (non-structural model) and a third probit model in which the only exogenous variable is the probability of default from the structural model (the model that contains only structural information). Thus, we examine the predictive power of the PD variable to explain corporate bankruptcy by integrating it in the non-structural model as an explanatory variable. If we find that the estimated coefficient of the variable PD (resulting from the structural model) is statistically different from zero, the probabilities of default obtained by the structural model in this case would contain additional information that complements that of accounting data. We use its coefficient to update the probabilities of default when the PD from the structural model changes. As to the choice of accounting variables and financial ratios used in the non-structural and hybrid models, we were faced with difficulties in the selection of variables given the scarcity of accounting and financial data on French listed companies that did default. To make a sound choice, we estimated the probit model on each variable accounting separately. This allows retaining the most significant ones.

4.3. Estimation results

4.3.1. Estimation of the probit model with different specifications

This section investigates the performance of three models: the hybrid model, the non-structural model and the model containing only structural information. We summarize the results of these estimations in Table 2. In Model 1, we use the information from the structural model by considering the mean PD (2 years) from the structural model as an explanatory variable. The coefficient of PD is 0.15%, and has the expected sign. It is a significant factor for predicting probabilities of default, with a p-value of less than 5% and a high corrected pseudo-R^2 (52.56%). In Model 2, we estimate the-non-structural model with 2 variables (the turnover and profitability ratio). Examination of Model 2 reveals that the non-structural specification largely outperforms the one using only information from the structural model (Model 1) in terms of its ability to explain corporate bankruptcy. The likelihood ratio is 17.63 for the non-structural model, versus 15.62 for the structural model with only PD as an exogenous variable (the corresponding values of R^2 are 59, 34% and 52.56%). (See Table 1.)

In Model 3, we estimate the hybrid model by adding the probabilities of default calculated from the structural model to the explanatory variables of Model 2. An analysis of the results reveals that the probabilities of default from the structural approach have an additional predictive power for corporate defaults than the firms' financial statements. We observe that the likelihood ratio increased from 17.63 for the non-structural model to 24.6 for the hybrid model (the corresponding values of corrected pseudo-R2 are 59.34% and 82.77%). Furthermore, the contribution of Model 3 relative to Model 2 is assessed by repeating this analysis in Models 4 and 5, but this time, by changing the variables used in the non-structural model and in Model 5. Model 5 estimates the hybrid model by adding default probabilities calculated from the structural model as explanatory variables to model 4.

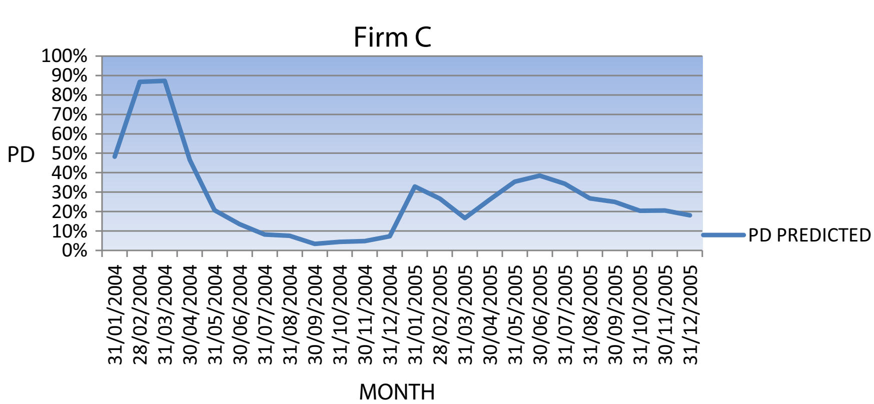

Fig. 2. Monthly default probabilities (2 years) of non-defaulting firms.

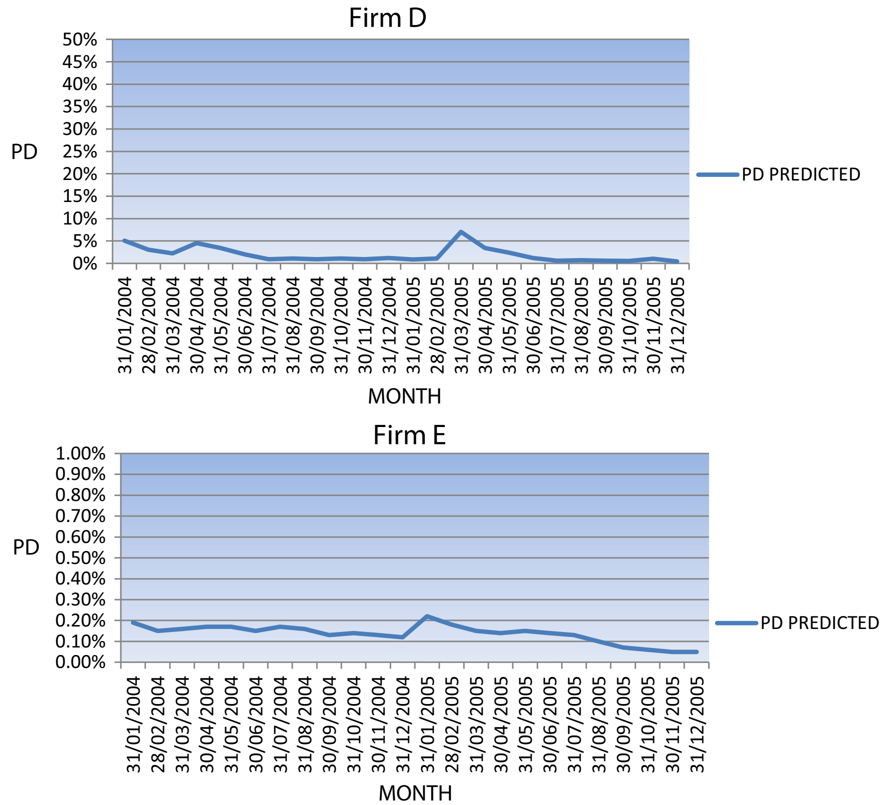

Fig. 3. Other PDs (2 years) of non-defaulting firms.

Analysis of the results gives us the same findings confirming the predictive power provided by incorporating the variable PD from the structural model to the non-structural model. Indeed, the likelihood ratio has increased from 18.24 for the non-structural model (model 4) to 21.23 for the hybrid model (the corresponding values of corrected pseudo-R2 are 61.37% and 71.43%).

4.3.2. Various tests

In Fig. 4, we reproduce the mean of the default probabilities for companies that did default of the five models estimated so far. The mean of the probabilities of default for the model containing only the structural information is 71.41%. This probability is maximized at 89.18% for the hybrid model with two accounting variables (turnover and profitability ratio) (Model 3). The same model but without the probabilities of default from the structural approach, comes in at 75.83%. This confirms the results from the previous section. (See Figs. 2 and 3.)

In Fig. 5, we reproduce the mean of the default probabilities for companies that did not default of the five models estimated. The mean of the probabilities of default for the model containing only the structural information is 15.73%. This probability is minimized at 5.56% for the hybrid model with two accounting variables (turnover and profitability ratio) (model 3). The same model but without the probabilities of default from the structural approach, has a mean of 15.3%. This also confirms the results from the previous section.

To investigate the performance of the hybrid models, we compare between the predictions of the probabilities of default of two of the five models used and the actual situation of firms. We found that the hybrid model dominates the other models. We again observe that the hybrid models are the best estimates of probability curves of defects.

5. Conclusion

Default led to the international global financial crisis in 2007–2008. Credit risk measurement is an area of great and renewed interest for both academicians and practitioners. Banks have to estimate defaults of their clients. In this paper, we investigate a major component of credit risk, the probability of default using a methodology in the spirit of Tudela and Young (2005). The methodology is applied to a sample of French companies whose shares are traded on the Stock Exchange Paris. This model has investigated the ability of hybrid models to

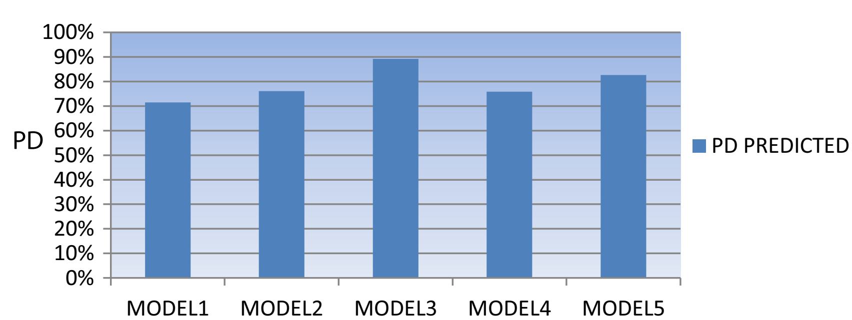

Fig. 4. Probabilities of default of firms that did default.

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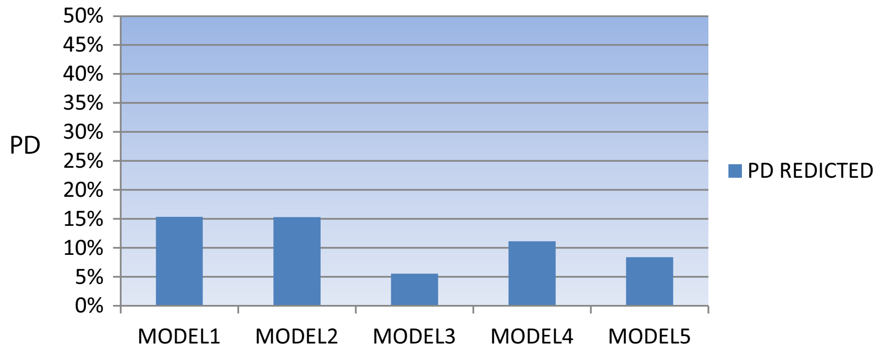

Fig. 5. Probabilities of default of firms that did not default.

calculate the default risk of UK companies by verifying whether combining the structural and the non-structural models into a hybrid model yields a better measure of the default risk than those obtained from structural and traditional non-structural models estimated separately.

We explain default deficiencies by estimating a probit model in which the explanatory variables are the estimated probabilities of default from the structural model, financial ratios and other accounting data.

The dependent variable is binary taking the value of 1 if the default occurs and 0 otherwise. We have also estimated a model with only accounting data as explanatory variables (non-structural model) and a third probit model in which the only exogenous variable is the probability of default from the structural model (the model that contains only structural information).

Thus, we have examined the predictive power of the default probabilities from the structural model to explain corporate bankruptcy by integrating it in the non-structural model as an explanatory variable.

Our results indicate that the predicted probabilities of default (PDs) contribute significantly to explaining default probabilities when they are included alongside the retained accounting variables. This confirms the results of the study of Tudela and Young (2005) and those of Dionne et al. (2005).

We note that the main limitation of our work was to fixing the default barrier. Thus, it would be interesting in future research to make endogenous default barrier levels and to estimate it by the maximum likelihood method that may increase the predictive ability model.

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